ENUNCIADO EJEMPLO 14

Un pendulo doble esta compuesto por dos particulas de masa m unidas entre si y a un punto fijo por sendos hilos sin masa de longitud l. El conjunto se halla en un plano vertical sujeto a su propio peso. Para describir el movimiento se emplearan los angulos phi y theta que forman los hilos con la vertical.

Paso 0. Reiniciación de las variables del sistema y llamada a los paquetes linalg, plots y plottools.

```
> restart:
> with(linalg):with(plots):with(plottools):
Warning, the protected names norm and trace have been redefined and unprotected

Warning, the name changecoords has been redefined

Warning, the name arrow has been redefined

> libname:="C:/",libname:
> with(mecapac3d):
```

Paso 1. Definimos las coordenadas generalizadas del sistema en una lista que se denominará cg.

```
> cg:=[phi,theta];
```

$$cg := [\phi, \theta]$$

Paso 2. Definición mediante variables de los elementos que forman el sistema mecánico.

```
> xg1:= [0,l/2*sin(phi),-l/2*cos(phi)]:
> xg2:=[0,l*sin(phi),-l*cos(phi)]:
> v1:=[varilla,xg1,rota(phi,1),M,l];
```

$$v1 := \left[varilla, \left[0, \frac{1}{2} l \sin(\phi), -\frac{1}{2} l \cos(\phi) \right], \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix}, M, l \right]$$

```
> p1:=[punto,0,l*sin(phi),-l*cos(phi),m];
```

$$p1 := [punto, 0, l \sin(\phi), -l \cos(\phi), m]$$

```
> xg3:= [0,l*sin(phi)+l/2*sin(theta),-l*cos(phi)-l/2*cos(theta)]:
> xg4:=[0,l*sin(phi)+l*sin(theta),-l*cos(phi)-l*cos(theta)]:
> v2:=[varilla,xg3,rota(theta,1),M,l];
```

$$v2 := \left[varilla, \left[0, l \sin(\phi) + \frac{1}{2} l \sin(\theta), -l \cos(\phi) - \frac{1}{2} l \cos(\theta) \right], \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}, M, l \right]$$

```
> p2:=[punto,0,l*sin(phi)+l*sin(theta),-l*cos(phi)-l*cos(theta),m];
```

$$p2 := [punto, 0, l \sin(\phi) + l \sin(\theta), -l \cos(\phi) - l \cos(\theta), m]$$

Paso 3. Definición de los elementos gráficos que definiran nuestro sistema de ejes.

```
> a1:=[angulo,[0,l/2*sin(phi),-l*cos(phi)],xg1,xg2,l/4]:
> a2:=[angulo,[0,l*sin(phi)+l/2*sin(theta),-l*cos(phi)-l*cos(theta)],xg3,x
  g4,l/4]:
> ejeZ:=[vector,[0,0,0],[0,0,l/2],blue]:
> ejeZ2:=[segmento,[0,0,-2*l],[0,0,0],blue]:
> TO := [texto,[0,1,0],"O"]:
> TZ := [texto,[0,1,l/2],"Z"]:
```

Paso 4. Definición de la variable sistema que agrupa en una lista todos los elementos anteriores.

```
> sistema:=[v1,p1,v2,p2,a1,a2,ejeZ,        ejeZ2,  TO,TZ]:
```

Paso 5. Obtención de la energía cinética del sistema mediante fT asignándola a la variable T.

```
> T:= fT(sistema);
```

$$T := \frac{1}{2} M \left(\frac{1}{4} l^2 \cos(\phi)^2 \phi1^2 + \frac{1}{4} l^2 \sin(\phi)^2 \phi1^2 \right) + \frac{1}{24} \phi1^2 M l^2 + \frac{1}{2} m (l^2 \cos(\phi)^2 \phi1^2 + l^2 \sin(\phi)^2 \phi1^2)$$
$$+ \frac{1}{2} M \left(\left(l \cos(\phi) \phi1 + \frac{1}{2} l \cos(\theta) \theta1 \right)^2 + \left(l \sin(\phi) \phi1 + \frac{1}{2} l \sin(\theta) \theta1 \right)^2 \right) + \frac{1}{24} \theta1^2 M l^2$$
$$+ \frac{1}{2} m ((l \cos(\phi) \phi1 + l \cos(\theta) \theta1)^2 + (l \sin(\phi) \phi1 + l \sin(\theta) \theta1)^2)$$

Paso 6. Obtención de la energía potencial del sistema mediante fV asignándola a la variable V.

```
> V:= fV(sistema);
```

$$V := -\frac{1}{2} M g l \cos(\phi) - m g l \cos(\phi) + M g \left(-l \cos(\phi) - \frac{1}{2} l \cos(\theta) \right) + m g (-l \cos(\phi) - l \cos(\theta))$$

Paso 7. Obtención de la lagrangiana como diferencia de energías entre la energía cinética y la potencial.

```
> L:=T-V;
```

$$L := \frac{1}{2} M \left(\frac{1}{4} l^2 \cos(\phi)^2 \phi1^2 + \frac{1}{4} l^2 \sin(\phi)^2 \phi1^2 \right) + \frac{1}{24} \phi1^2 M l^2 + \frac{1}{2} m (l^2 \cos(\phi)^2 \phi1^2 + l^2 \sin(\phi)^2 \phi1^2)$$
$$+ \frac{1}{2} M \left(\left(l \cos(\phi) \phi1 + \frac{1}{2} l \cos(\theta) \theta1 \right)^2 + \left(l \sin(\phi) \phi1 + \frac{1}{2} l \sin(\theta) \theta1 \right)^2 \right) + \frac{1}{24} \theta1^2 M l^2$$
$$+ \frac{1}{2} m ((l \cos(\phi) \phi1 + l \cos(\theta) \theta1)^2 + (l \sin(\phi) \phi1 + l \sin(\theta) \theta1)^2) + \frac{1}{2} M g l \cos(\phi) + m g l \cos(\phi)$$
$$- M g \left(-l \cos(\phi) - \frac{1}{2} l \cos(\theta) \right) - m g (-l \cos(\phi) - l \cos(\theta))$$

Paso 8. Obtención de las ecuaciones de lagrange para las dos coordenadas generalizadas mediante el operador Ec_lag

```
> ecua:=ec_lag();
```

$$
\begin{aligned}
ecua := \Bigg[& \\
& \frac{1}{2} M \left(\frac{1}{2} l^2 \cos(\phi(t))^2 \left(\frac{d^2}{dt^2} \phi(t) \right) + \frac{1}{2} l^2 \sin(\phi(t))^2 \left(\frac{d^2}{dt^2} \phi(t) \right) \right) + \frac{1}{12} \left(\frac{d^2}{dt^2} \phi(t) \right) M l^2 \\
& + \frac{1}{2} m \left(2 l^2 \cos(\phi(t))^2 \left(\frac{d^2}{dt^2} \phi(t) \right) + 2 l^2 \sin(\phi(t))^2 \left(\frac{d^2}{dt^2} \phi(t) \right) \right) \\
& + \frac{1}{2} M \left(2 \left(-l \sin(\phi(t)) \left(\frac{d}{dt} \phi(t) \right)^2 + l \cos(\phi(t)) \left(\frac{d^2}{dt^2} \phi(t) \right) - \frac{1}{2} l \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right)^2 \right. \right. \\
& \left. \left. + \frac{1}{2} l \cos(\theta(t)) \left(\frac{d^2}{dt^2} \theta(t) \right) \right) l \cos(\phi(t)) \right. \\
& - 2 \left(l \cos(\phi(t)) \left(\frac{d}{dt} \phi(t) \right) + \frac{1}{2} l \cos(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \right) l \sin(\phi(t)) \left(\frac{d}{dt} \phi(t) \right) \\
& + 2 \left(l \cos(\phi(t)) \left(\frac{d}{dt} \phi(t) \right)^2 + l \sin(\phi(t)) \left(\frac{d^2}{dt^2} \phi(t) \right) + \frac{1}{2} l \cos(\theta(t)) \left(\frac{d}{dt} \theta(t) \right)^2 + \frac{1}{2} l \sin(\theta(t)) \left(\frac{d^2}{dt^2} \theta(t) \right) \right. \\
& \left. \left. \right) l \sin(\phi(t)) + 2 \left(l \sin(\phi(t)) \left(\frac{d}{dt} \phi(t) \right) + \frac{1}{2} l \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \right) l \cos(\phi(t)) \left(\frac{d}{dt} \phi(t) \right) \right) \\
& + \frac{1}{2} m \left(2 \left(-l \sin(\phi(t)) \left(\frac{d}{dt} \phi(t) \right)^2 + l \cos(\phi(t)) \left(\frac{d^2}{dt^2} \phi(t) \right) - l \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right)^2 \right. \right.
\end{aligned}
$$

$$+ l\cos(\theta(t))\left(\frac{d^2}{dt^2}\theta(t)\right)\Bigg)\, l\cos(\phi(t))$$

$$- 2\left(l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right)$$

$$+ 2\left(l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right)^2 + l\sin(\phi(t))\left(\frac{d^2}{dt^2}\phi(t)\right) + l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)^2 + l\sin(\theta(t))\left(\frac{d^2}{dt^2}\theta(t)\right)\right) l$$

$$\sin(\phi(t)) + 2\left(l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right)\Bigg)$$

$$-\frac{1}{2}M\left(-2\left(l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + \frac{1}{2}l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right)\right.$$

$$\left. + 2\left(l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + \frac{1}{2}l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right)\right)$$

$$-\frac{1}{2}m\left(-2\left(l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right)\right.$$

$$\left. + 2\left(l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right)\right) + \frac{3}{2}M g l\sin(\phi(t))$$

$$+ 2 m g l\sin(\phi(t)),$$

$$\frac{1}{2}M\left(\left(-l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right)^2 + l\cos(\phi(t))\left(\frac{d^2}{dt^2}\phi(t)\right) - \frac{1}{2}l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)^2\right.\right.$$

$$\left.\left. + \frac{1}{2}l\cos(\theta(t))\left(\frac{d^2}{dt^2}\theta(t)\right)\right) l\cos(\theta(t))\right.$$

$$\left. - \left(l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + \frac{1}{2}l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right.$$

$$+\left(l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right)^2 + l\sin(\phi(t))\left(\frac{d^2}{dt^2}\phi(t)\right) + \frac{1}{2}l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)^2 + \frac{1}{2}l\sin(\theta(t))\left(\frac{d^2}{dt^2}\theta(t)\right)\right)$$

$$l\sin(\theta(t)) + \left(l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + \frac{1}{2}l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)$$

$$+\frac{1}{12}\left(\frac{d^2}{dt^2}\theta(t)\right) M l^2$$

$$+\frac{1}{2}m\left(2\left(-l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right)^2 + l\cos(\phi(t))\left(\frac{d^2}{dt^2}\phi(t)\right) - l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)^2\right.\right.$$

$$\left.\left. + l\cos(\theta(t))\left(\frac{d^2}{dt^2}\theta(t)\right)\right) l\cos(\theta(t))\right.$$

$$-2\left(l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)$$

$$+2\left(l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right)^2 + l\sin(\phi(t))\left(\frac{d^2}{dt^2}\phi(t)\right) + l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)^2 + l\sin(\theta(t))\left(\frac{d^2}{dt^2}\theta(t)\right)\right) l$$

$$\sin(\theta(t)) + 2\left(l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)$$

$$-\frac{1}{2}M\left(-\left(l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + \frac{1}{2}l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right.$$

$$\left. + \left(l\sin(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + \frac{1}{2}l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right)$$

$$-\frac{1}{2}m\left(-2\left(l\cos(\phi(t))\left(\frac{d}{dt}\phi(t)\right) + l\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right) l\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)\right.$$

$$+ 2\left(l \sin(\phi(t)) \left(\frac{d}{dt}\phi(t) \right) + l \sin(\theta(t)) \left(\frac{d}{dt}\theta(t) \right) \right) l \cos(\theta(t)) \left(\frac{d}{dt}\theta(t) \right) + \frac{1}{2} M g l \sin(\theta(t))$$

$$+ m g l \sin(\theta(t))$$

Paso 9. Asignación de valores numéricos a los parámetros que queden sun asignar para poder proceder a la integración numérica.

```
> m:=1; M:=9;g:=9.8;l:=3;m2:=0.001:
```

$$m := 1$$

$$M := 9$$

$$g := 9.8$$

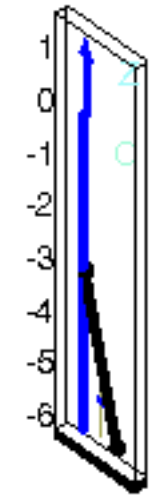
$$l := 3$$

Paso 10. Integración numérica del problema mediante la función fint asignando el resultado a la variable res.

```
> res:= fint([evalf(0.),evalf(0.),evalf(0.2),evalf(0.2)]):
```

Paso 11. Procedemos a realizar una animación del movimiento del conjunto por medio de la función dibu3.

```
> dibu3(2,70);
```


1
0
-1
-2
-3
-4
-5
-6